MATHEMATICS METHODS

MAWA Semester 2 (Units 3 and 4) Examination 2016

Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2016**

Section One: Calculator-free (54 Marks)

Question 1(a)

Solution	
$\ln m = \frac{3}{2} \Rightarrow m = e^{\frac{3}{2}}$	
Marking key/mathematical behaviours	**Marks**
• identifies correct base • determines correct power	1 1

Question 1(b)

Solution	
$\log[(m+3)m] = 1$ $(m+3)m = 10^1$ $m^2 + 3m - 10 = 0$ $(m+5)(m-2) = 0$ $m = -5$ or 2 but since m has to be greater than zero, $m = 2$ is the only solution.	
Marking key/mathematical behaviours	**Marks**
• applies logarithmic rule for a product correctly • recognises base 10 • creates equation with correct trinomial • solves equation correctly giving the correct value of m	1 1 1 1

Question 2(a) (i)

Solution	
$\frac{dy}{dx} = \frac{(6x^4 - x^3 + e)(4e^x) - (4e^x)(24x^3 - 3x^2)}{\left(6x^4 - x^3 + e\right)^2}$	
Marking key/mathematical behaviours	**Marks**
• differentiates the 1st term on numerator correctly • differentiates the 2nd term on numerator correctly • squares factor on denominator	1 1 1

Question 2(a)(ii)

Solution

$$\frac{dy}{dx} = \frac{d}{dx}[\ln\left(5x^3+3\right) - \ln(\sin(x)]$$
$$= \frac{d}{dx}[\ln\left(5x^3+3\right)] - \frac{d}{dx}[\ln(\sin(x)]$$
$$= \frac{15x^2}{\left(5x^3+3\right)} - \frac{\cos(x)}{\sin(x)}$$

Marking key/mathematical behaviours	Marks
• applies correctly logarithmic rule for quotients	1
• differentiates correctly 1st term	1
• differentiates correctly 2nd term	1

Question 2(b)

Solution

Let $u = x^2 - \cos\left(x\right) \Rightarrow \frac{du}{dx} = 2x + \sin\left(x\right)$ and $\frac{dy}{du} = \frac{e^u}{2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{e^u}{2} \times (2x + \sin\left(x\right)) = \frac{e^{x^2 - \cos(x)}}{2}(2x + \sin\left(x\right))$$

Marking key/mathematical behaviours	Marks
• differentiates correctly to determine1st factor in chain rule	1
• differentiates correctly to determine 2nd factor in chain rule	1
• expresses $\frac{dy}{dx}$ in terms of x	1

Question 3(a)

Solution Discrete random variable	
Marking key/mathematical behaviours	Marks
• determines correct category	1

Question 3(b)

Solution Non-random variable	
Marking key/mathematical behaviours	Marks
• determines correct category	1

Question 3(c)

Solution Continuous random variable	
Marking key/mathematical behaviours	Marks
• determines correct category	1

Question 4

Solution	
$k\int_0^1 x - \frac{x^3}{3}\ dx = 1$ $k\left[\frac{x^2}{2} - \frac{x^4}{12}\right]_0^1 = 1$ $k\left[\frac{1}{2} - \frac{1}{12}\right] = 1 \quad \Rightarrow \quad k = \frac{12}{5}$	
Marking key/mathematical behaviours	Marks
• sets up integral and equates to one • integrates correctly • evaluates integral correctly • calculates the value of k	1 1 1 1

Question 5

Solution	
$p(1-p) = \left(\frac{\sqrt{3}}{4}\right)^2 = \frac{3}{16}$ $16p^2 - 16p + 3 = 0$ $(4p-1)(4p-3) = 0 \quad \Rightarrow \quad p = \frac{1}{4}$ or $p = \frac{3}{4}$	
Marking key/mathematical behaviours	Marks
• sets up equation using variance of a Bernoulli distribution • derives quadratic equation • factorises trinomial • solves correctly for p	1 1 1 1

Question 6(a)

Solution Function is valid for $x > -3$	
Marking key/mathematical behaviours	Marks
• correctly states the values of *x* for which the function is valid	1

Question 6(b)

Solution $\frac{dy}{dx} = \frac{2}{2x+6} = 4 \Rightarrow \frac{2x+6}{2} = \frac{1}{4} \Rightarrow x+3 = \frac{1}{4} \Rightarrow x = -2.75$	
Marking key/mathematical behaviours	Marks
• differentiates correctly • solves equation correctly	1 1

Question 7(a)

Solution

y	0	1	2	3	4
$P(Y = y)$	**0**	***k***	$4k$	***9k***	***16k***

Marking key/mathematical behaviours	Marks
• correctly completes two values • correctly completes 4 values	1 1

Question 7(b)

Solution $k + 4k + 9k + 16k = 1$ $30k = 1 \quad \Rightarrow \quad k = \frac{1}{30}$	
Marking key/mathematical behaviours	Marks
• sums probabilities equal to one • correctly solves equation for k	1 1

Question 8

Solution

$$f(x) = \int f'(x)\,dx$$
$$= \int 2xe^{3x^2-1}\,dx$$
$$= \frac{1}{3}e^{3x^2-1} + c$$

since $f(0) = 0$:

$$0 = \frac{1}{3}e^{-1} + c$$
$$c = -\frac{1}{3e}$$
$$f(x) = \frac{1}{3}e^{3x^2-1} - \frac{1}{3e}$$

Marking key/mathematical behaviours	Marks
• determines indefinite integral	1
• substitutes initial conditions to calculate the constant c	1
• states f(x)	1

Question 9 (a)(i)

Solution

$$\hat{p} = \frac{20}{100} = \frac{1}{5}$$

Marking key/mathematical behaviours	Marks
• determines the proportion	1

Question 9(a)(ii)

Solution

$$E = 2 \times \sqrt{\frac{\frac{1}{5}(1-\frac{1}{5})}{100}}$$
$$= 2 \times \sqrt{\frac{4}{2500}}$$
$$= 2 \times \frac{2}{50}$$
$$= 0.08$$

95% CI is $(0.12, 0.28)$

Marking key/mathematical behaviours	Marks
• substitutes values for z, n and p	1
• simplifies square root	1
• simplifies E	1
• states interval	1

Question 9(b)

Solution

$$E = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$$

68% CI is $\left(m - \sqrt{\frac{m(1-m)}{n_1}}, m + \sqrt{\frac{m(1-m)}{n_1}}\right)$

Marking key/mathematical behaviours	Marks
• determines E	1
• states confidence interval.	1

Question 9(c)(i)

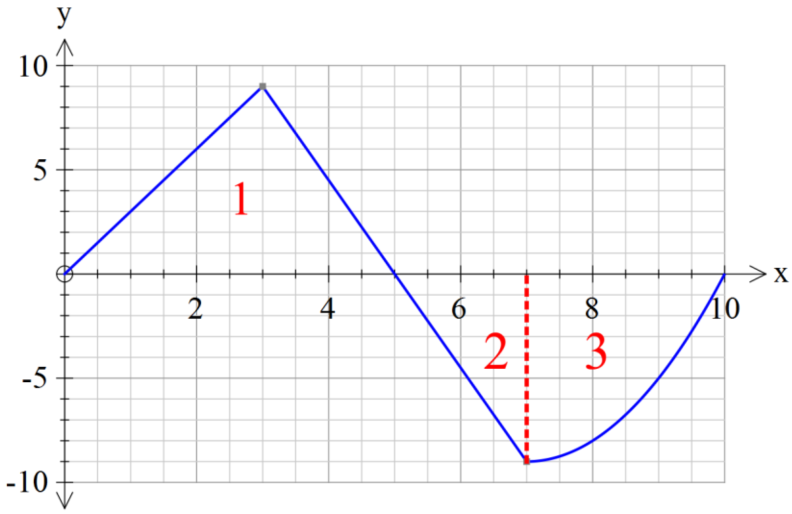
Solution	
n_2 is larger than n_1 To increase confidence a larger interval is required for a stable sample size. Increasing n reduces the standard error and thus the interval can remain the same.	
Marking key/mathematical behaviours	Marks
• states n_2 is larger with reason • states correct reason	1 1

Question 9(c)(ii)

Solution	
$E_1 = 1 \times \sqrt{\frac{m(1-m)}{n_1}}$ $E_2 = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$ Same interval so $E_1 = E_2$ $\sqrt{\frac{m(1-m)}{n_1}} = 1.5 \times \sqrt{\frac{m(1-m)}{n_2}}$ $\frac{m(1-m)}{n_1} = (1.5)^2 \frac{m(1-m)}{n_2}$ $\frac{n_2}{n_1} = 2.25$ $n_2 = 2.25 n_1$	
Marking key/mathematical behaviours	Marks
• equates E_1 and E_2 • squares both sides • states relationship	1 1 1

Question 10(a)

Solution



$$\int_0^5 f(x)dx = \text{area of triangle (Area 1)}$$

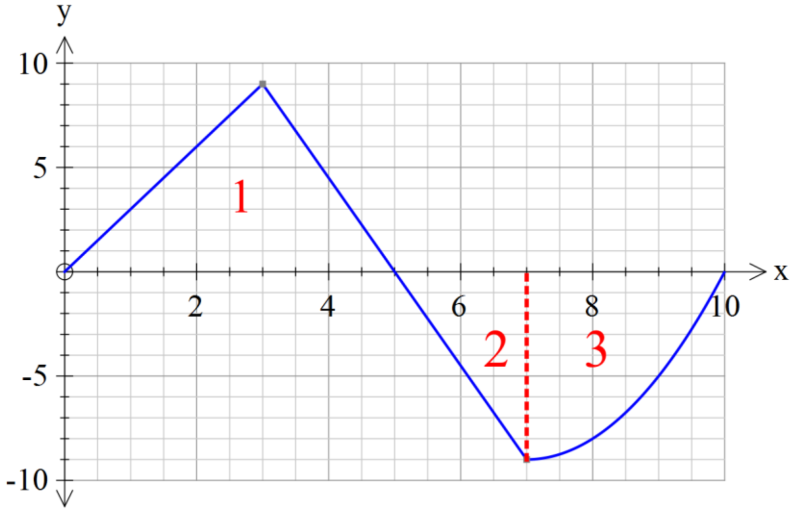
$$= \frac{1}{2} \times 5 \times 9$$

$$= 22\frac{1}{2}$$

Marking key/mathematical behaviours	Marks
• identifies integral as area of correct triangle	1
• determines integral.	1

Question 10(b)

Solution



$$\text{Area } 2 = \frac{1}{2} \times 2 \times 9$$
$$= 9 \text{ square units}$$

$$\text{Area } 3 = 50 - 22\frac{1}{2} - 9$$
$$= 18\frac{1}{2} \text{ square units}$$

$$\int_7^{10} f(x)dx = -\text{ Area } 3$$
$$= -18\frac{1}{2}$$

Marking key/mathematical behaviours	Marks
• calculates area 2	1
• calculates area 3	1
• determines integral	1